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New insights on lubrication theory for compressible fluids

Andreas Almqvist[a,*], Evgeniya Burtseva[b], Francesc Pérez-Ráfols[a], Peter Wall[b]

[a] Machine Elements, Luleå University of Technology, Luleå 97187, Sweden
[b] Mathematical Sciences, Luleå University of Technology, Luleå 97187, Sweden

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ABSTRACT

The fact that the film is thin is in lubrication theory utilised to simplify the full Navier–Stokes system of equations. For incompressible and iso-viscous fluids, it turns out that the inertial terms are small enough to be neglected. However, for a compressible fluid, we show that the influence of inertia depends on the (constitutive) density-pressure relationship and may not always be neglected. We consider a class of iso-viscous fluids obeying a power-law type of compressibility, which in particular includes both incompressible fluids and ideal gases. We show by scaling and asymptotic analysis, that the degree of compressibility determines whether the terms governing inertia may or may not be neglected. For instance, for an ideal gas, the inertial terms remain regardless of the film height-to-length ratio. However, by means of a specific modified Reynolds number that we define we show that the magnitudes of the inertial terms rarely are large enough to be influential. In addition, we consider fluids obeying the well-known Dowson and Higginson density-pressure relationship and show that the inertial terms can be neglected, which allows for obtaining a Reynolds type of equation. Finally, some numerical examples are presented in order to illustrate our theoretical results.

1. Introduction

A central part in classical lubrication theory is the modelling and analysis of fluid flow in thin gaps. Examples of where fluid flow in thin gaps take place are between the ball and the raceway in a bearing, between the contacting surfaces in a seal, between the eye and the contact lens, in our joints and when a water film is generated between a car's tyre and the road surface. Most of the theoretical results in this field may be seen as developments of the classical work by Reynolds (1886), to which complexity has been added. In particular, compressible flow of iso-viscous fluids have been thoroughly studied, as it is obvious that compressibility plays a role in systems such as in static metal-to-metal seals and in oil- and gas lubricated hydrodynamic slider bearings, see e.g. (Agrawal, 1997; Brunetière & Tournerie, 2006; Childs, 1993; Dupuy et al., 2016; Dupuy, Bou-Saïd, & Tichy, 2015; Fuller, 1969; Marušić-Paloka & Starčević, 2010; Pérez-Ráfols, Wall, & Almqvist, 2018). In this paper, we present a rigorous asymptotic analysis of the Navier–Stokes equations including the effect of compressibility, leading to a thin film approximation which differs from the classical result and questions its applicability.

* Corresponding author.
E-mail address: andreas.almqvist@ltu.se (A. Almqvist).

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